

## Progression in written addition methods

### Foundation 2

Children find the total of objects in 2 groups by counting all of them.

- Through practical activities, using fingers and through discussion they will begin to use the vocabulary involved in addition.

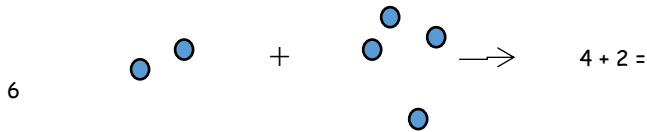


'You have five apples and I have two apples. How many apples altogether?'

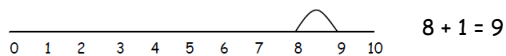
- They will record pictorially then numerically  
 $5 + 2 = 7$  apples

Children add 2 single digit numbers by counting on.

- Through practical activities, children to begin counting on, starting from the highest number.



- Using a number line counting from the biggest number.



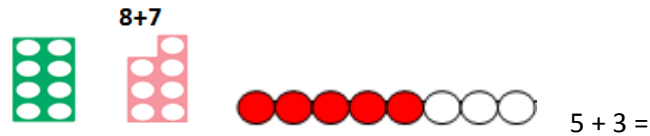
Children will find one more than a given number.

- Through songs, rhymes and practical activities children develop a sense of number.
- Children will use number line to find one more than a given number.

### Year 1

Children, read, write and interpret mathematical statements involving addition (+) and the equals (=).

- Through practical activities, using rods, cubes, numicom, number beads, number lines and 100 squares.



Children add one-digit and two-digit numbers within 20, including zero

- Using practical equipment children combine groups, counting from the largest.
- Using a number line to add two numbers together, encouraging children to start from the largest number.
- using number bonds and related addition facts within 20 which have been learned.
- Using a 100 square to add in steps of 1 or 10.

12 | 13 | 14 | 15       $13 + 2 = 15$

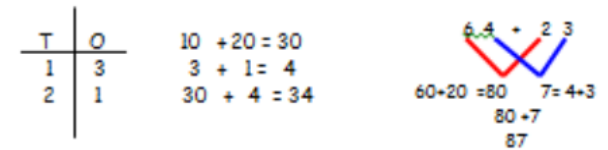
Children solve missing number problems by counting on from the given number. eg  $10 + \underline{\quad} = 16$



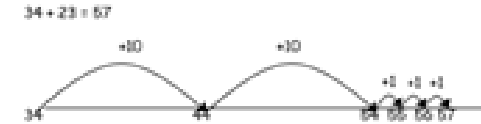
### Year 2

Children solve problems with addition using concrete objects and pictorial representations. Children will learn to add -:

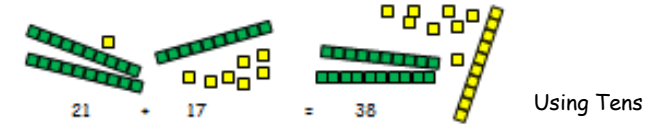
- A two digit number and ones
  - A two digit number and tens
  - Two two-digit numbers
  - Three one-digit numbers
- Using place value knowledge children combine Tens and Ones to add.



- Using an empty number line to add two-digit numbers.

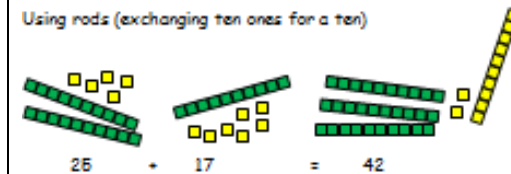


- Using Tens and Ones apparatus children add by combining groups, counting from the largest.



and apparatus children add by combining groups, where 10 ones are exchanged for a Ten.

Using rods (exchanging ten ones for a ten)



## Progression in written addition methods

Year 3	Year 4	Year 5	Year 6																																																																																																																								
<p>Add numbers with up to three digits, using formal written method of columnar addition</p> <p><b>Stage 1 no exchange and expanded</b></p> $\begin{array}{r} 21 \quad 20+1 \\ +17 \quad 10+7 \\ \hline 38 \quad 30+8 \end{array}$ <p><b>Stage 2 with exchange and expanded</b></p> $\begin{array}{r} 25 \quad 20+5 \\ +17 \quad 10+7 \\ \hline 42 \quad 30+12=42 \end{array}$ <p><b>Stage 3 begin to represent as vertical alongside expanded</b></p> $\begin{array}{r} 67 \quad (60+7) \\ +24 \quad (20+4) \\ \hline 80+11=91 \end{array} \quad \begin{array}{r} +24 \\ 11(7+4) \\ \hline 80(60+20) \\ 91 \end{array}$ <p><b>Stage 4 more than two numbers recorded vertically</b></p> $\begin{array}{r} 24 \\ 67 \\ +12 \\ \hline 13(4+7+2) \\ 90(20+60+10) \\ 103(90+13) \end{array}$	<p>Add numbers with up to 4 digits using the formal written methods of columnar addition. Carry below the line and cross off when added into the calculation</p> $\begin{array}{r} 625 \quad 783 \\ +48 \quad +42 \\ \hline 673 \quad 825 \\ \cancel{1} \quad \cancel{1} \end{array}$ <div style="text-align: center;"> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr><td> </td><td>2</td><td>5</td><td>6</td></tr> <tr><td>+</td><td>1</td><td>2</td><td>7</td></tr> <tr><td colspan="4" style="border-top: 1px solid black;"></td></tr> <tr><td> </td><td>3</td><td>8</td><td>3</td></tr> <tr><td colspan="4" style="border-top: 1px solid black;"></td></tr> <tr><td> </td><td> </td><td style="text-align: center;"><math>\cancel{1}</math></td><td> </td></tr> </table>   <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr><td> </td><td>5</td><td>2</td><td>0</td><td>9</td></tr> <tr><td>+</td><td>3</td><td>1</td><td>9</td><td>2</td></tr> <tr><td colspan="5" style="border-top: 1px solid black;"></td></tr> <tr><td> </td><td>8</td><td>4</td><td>0</td><td>1</td></tr> <tr><td colspan="5" style="border-top: 1px solid black;"></td></tr> <tr><td> </td><td> </td><td style="text-align: center;"><math>\cancel{1}</math></td><td style="text-align: center;"><math>\cancel{1}</math></td><td> </td></tr> </table> </div>		2	5	6	+	1	2	7						3	8	3							$\cancel{1}$			5	2	0	9	+	3	1	9	2							8	4	0	1								$\cancel{1}$	$\cancel{1}$		<p>Add whole numbers with more than 4 digits, including using formal methods (columnar)</p> $\begin{array}{r} 23587 \\ +1475 \\ \hline 25062 \\ 50124 \\ \hline 121 \end{array}$ <div style="text-align: center;"> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr><td> </td><td>5</td><td>2</td><td>0</td><td>9</td></tr> <tr><td>+</td><td>3</td><td>1</td><td>9</td><td>2</td></tr> <tr><td colspan="5" style="border-top: 1px solid black;"></td></tr> <tr><td> </td><td>8</td><td>4</td><td>0</td><td>1</td></tr> <tr><td colspan="5" style="border-top: 1px solid black;"></td></tr> <tr><td> </td><td> </td><td style="text-align: center;"><math>\cancel{1}</math></td><td style="text-align: center;"><math>\cancel{1}</math></td><td> </td></tr> </table> </div> <p>Begin to add two or more decimal fractions with up to three digits and the same number of decimal places;</p> <div style="text-align: center;"> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr><td> </td><td>3</td><td>8</td><td>.</td><td>3</td><td>6</td></tr> <tr><td>+</td><td>2</td><td>7</td><td>.</td><td>9</td><td>5</td></tr> <tr><td colspan="6" style="border-top: 1px solid black;"></td></tr> <tr><td> </td><td>6</td><td>6</td><td>.</td><td>3</td><td>1</td></tr> <tr><td colspan="6" style="border-top: 1px solid black;"></td></tr> <tr><td> </td><td style="text-align: center;"><math>\cancel{1}</math></td><td style="text-align: center;"><math>\cancel{1}</math></td><td> </td><td style="text-align: center;"><math>\cancel{1}</math></td><td> </td></tr> </table> </div> <p>Know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m - 280 cm. when working with decimals, always make each number have the same amount of digits to the right of the decimal point</p> $3.7 + 2.35 = \begin{array}{r} 3.70 \\ +2.35 \\ \hline 6.05 \\ \cancel{1} \end{array}$		5	2	0	9	+	3	1	9	2							8	4	0	1								$\cancel{1}$	$\cancel{1}$			3	8	.	3	6	+	2	7	.	9	5								6	6	.	3	1								$\cancel{1}$	$\cancel{1}$		$\cancel{1}$		<p>There is an expectation that children will continue to practise and use the formal written method for larger numbers and decimals and use these methods when solving problems, when appropriate (see previous year's guidance for methods).</p>
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<p>Add fractions with the same denominator within one whole</p> $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$	<p>Add fractions of the same denominators</p> $1\frac{1}{6} + \frac{4}{6} = 1\frac{5}{6}$	<p>Add fractions with same denominators and multiples of the same number</p> $1\frac{1}{6} + \frac{4}{6} = 1\frac{5}{6}$ $\frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$	<p><math>2\frac{1}{4} + 1\frac{1}{6} = 1\frac{3}{12} + 1\frac{2}{12} = 3\frac{5}{12}</math></p> <p>Pupils should add fractions with different denominators and mixed numbers using the concept of equivalent fractions.</p>																																																																																																																								

## Progression in written subtraction methods

### Foundation 2

Children will engage in a variety of counting songs, rhymes and practical activities to develop a sense of number.

Children will find one less than a given number.

- In practical activities, using objects and fingers they will begin to use the vocabulary involved in subtraction



'You have five apples and I eat one apple. How many apples left?'

- They will record pictorially then numerically  $5 - 1 = 4$  apples

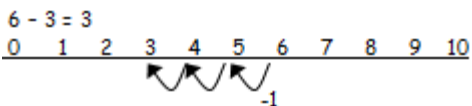
Children subtract from 2 single digit numbers, by counting back to find the answer

- Using objects then pictures, children subtract a single digit number



$5 \text{ subtract } 3 = 2$

- Using a number line children count back below the line to show subtraction.

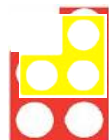


### Year 1

Children read, write & interpret mathematical statements involving subtraction (-) & equals (=).

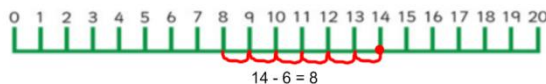
- Through practical activities, using rods, cubes, numicom, number beads, number lines and 100 squares.

$5 - 3 = 2$



Children subtract one-digit & two-digit numbers to 20, including zero.

- Using a number line to subtract a number, counting back below the line.



- using number bonds and related addition facts within 20 which have been learned.
- Using a 100 square to add in steps of 1 or 10.

12	<del>13</del>	<del>14</del>	<del>15</del>	$15 - 2 = 13$	$66 - 20 = 46$	45	46	47
55	56	57				65	66	67

Children begin to find the difference using subtraction.

### Year 2

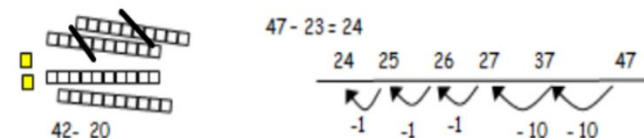
Children subtract numbers using concrete objects and pictorial representations. Children will learn to subtract -:

- A two digit number and ones
- A two digit number and tens
- Two two-digit numbers
- Using knowledge of addition and subtraction families and the inverse relationship of addition and subtraction.

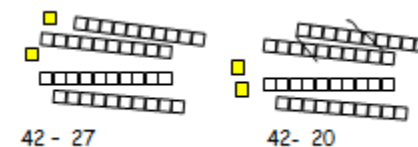
$3 + 2 = 5$       $2 + 3 = 5$      Number families  
 $5 - 2 = 3$       $5 - 3 = 2$

$48 + 36 = 84$      so      $84 - 36 = 48$      Inverse

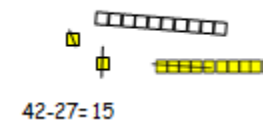
- Using place value knowledge children subtract Tens and Ones.



- Using Tens and Ones apparatus children subtract by removing rods where 10 ones are exchanged for a Ten.



subtract 7 by exchanging a ten rod for 10 units



## Progression in written subtraction methods

Year 3	Year 4	Year 5	Year 6																												
<p>Subtract numbers with up to three digits, using formal written method of columnar subtraction</p> <p><b>Stage 1 - expanded with no exchange</b></p> $\begin{array}{r} 89 = 80 + 9 \\ - 57 = 50 + 7 \\ \hline 30 + 2 = 32 \end{array}$ <p><b>Stage 2 - expanded with exchange</b></p> <p style="text-align: center;"><i>Step 1</i></p> <p>The calculation should be read as e.g. 1 minus/take/subtract 6</p> $\begin{array}{r} 71 \\ - 46 \\ \hline 25 \end{array}$ <p style="text-align: center;"><i>Step 2</i></p> $\begin{array}{r} 70 + 1 \\ - 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$ <p style="text-align: center;"><i>when confident</i></p> <p style="text-align: center;"><i>Step 3</i></p> $\begin{array}{r} 754 \\ - 86 \\ \hline 668 \end{array}$ <p style="text-align: center;"><i>Step 1</i></p> $\begin{array}{r} 700 + 50 + 4 \\ - 80 + 6 \\ \hline 600 + 60 + 8 = 668 \end{array}$ <p style="text-align: center;"><i>Step 2</i></p> $\begin{array}{r} 700 + 40 + 14 \\ - 80 + 6 \\ \hline 600 + 60 + 8 = 668 \end{array}$ <p style="text-align: center;"><i>Step 3</i></p> $\begin{array}{r} 600 + 140 + 14 \\ - 80 + 6 \\ \hline 600 + 60 + 8 = 668 \end{array}$ <p style="text-align: center;"><i>Step 4</i></p> $\begin{array}{r} 600 + 140 + 14 \\ - 70 + 10 + 6 \\ \hline 600 + 60 + 8 = 668 \end{array}$	<p>Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate</p> $\begin{array}{r} 6141 \\ - 86 \\ \hline 668 \end{array}$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>5</td><td>6<sup>5</sup></td><td>12</td></tr> <tr><td>-</td><td>3</td><td>1</td><td>9</td></tr> <tr><td></td><td>2</td><td>4</td><td>3</td></tr> </table> <p><b>NOTE:</b> In both examples, we have 'exchanged' one of the six tens for ten ones. •</p> <p>Children will also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds; know that decimal points should line up under each other</p> <p><b>£3.50- £1.67</b></p> $\begin{array}{r} 1.67 - 0.3 \\ - 0.30 \\ \hline 1.37 \end{array}$	5	6 <sup>5</sup>	12	-	3	1	9		2	4	3	<p>Subtract whole numbers with more than 4 digits, including using formal written method (columnar subtraction)</p> <p>Use compact ('decomposition') method and the vocabulary 'exchange'</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>8<sup>7</sup></td><td>1<sup>6</sup></td><td>.</td><td>3<sup>2</sup></td><td>1<sup>0</sup></td></tr> <tr><td>-</td><td>1</td><td>9</td><td>.</td><td>0</td><td>4</td></tr> <tr><td></td><td>6</td><td>7</td><td>.</td><td>2</td><td>6</td></tr> </table> <p>In this example, we have 'exchanged' one of the three tenths for ten hundredths</p>	8 <sup>7</sup>	1 <sup>6</sup>	.	3 <sup>2</sup>	1 <sup>0</sup>	-	1	9	.	0	4		6	7	.	2	6	<p><b>As per Year 5</b></p> <p>Subtract fractions with different denominators</p> $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$
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<p>Subtract fractions with the same denominator within one whole</p> $\frac{6}{7} - \frac{1}{7} = \frac{5}{7}$	<p>Subtract fractions with the same denominator</p> $\frac{1}{6} - \frac{4}{6} = \frac{3}{6}$	<p>Subtract fractions with the same denominator and multiples of the same number</p> $\frac{1}{4} - \frac{1}{8} = \frac{2}{8} - \frac{1}{8} = \frac{1}{8}$	<p>Subtract fractions with different denominators</p> $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$																												